

# $\Delta K$ , $\Lambda K$ , and $\Sigma K$ states in the extended chiral SU(3) quark model

F. Huang<sup>1,2,3</sup> and Z.Y. Zhang<sup>2</sup>

<sup>1</sup>*CCAST (World Laboratory), P.O. Box 8730, Beijing 100080, PR China*

<sup>2</sup>*Institute of High Energy Physics, P.O. Box 918-4, Beijing 100049, PR China\**

<sup>3</sup>*Graduate School of the Chinese Academy of Sciences, Beijing, PR China*

## Abstract

By use of the resonating group method, the  $\Delta K$ ,  $\Lambda K$ , and  $\Sigma K$  states are further dynamically studied in the extended chiral SU(3) quark model based on our previous work. Similar to the results given by the original chiral SU(3) quark model, the calculated results here still show that the interactions of  $\Delta K$  with isospin  $I = 1$  and  $\Sigma K$  with isospin  $I = 1/2$  are attractive, which can consequently lead to  $\Delta K$  and  $\Sigma K$  quasibound states. When the channel coupling of  $\Lambda K$  and  $\Sigma K$  is considered, the calculated phase shifts show a sharp resonance between the thresholds of these two channels with spin-parity  $J^P = 1/2^-$ .

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\* Mailing address.

As we know, the non-perturbative QCD (NPQCD) effect is very important in the light quark system. Since it is difficult to seriously solve the NPQCD effect, QCD-inspired models are still needed to connect the theoretical results and the experimental observables. Among these models, the chiral SU(3) quark model has been quite successful in reproducing the energies of the baryon ground states, the binding energy of deuteron, the nucleon-nucleon ( $NN$ ) scattering phase shifts, and the nucleon-hyperon ( $NY$ ) cross sections. Recently, we extended the chiral SU(3) quark model to study the baryon-meson systems by solving a resonating group method (RGM) equation. In Refs. [1, 2], we studied the kaon-nucleon ( $KN$ ) scattering phase shifts, and a satisfactory agreement with the experiment is obtained. Further, in Refs. [2, 3], we dynamically studied the structures of the  $\Delta K$ ,  $\Lambda K$ , and  $\Sigma K$  states. Our results show that the  $\Delta K$  with isospin  $I = 1$  and the  $\Sigma K$  with isospin  $I = 1/2$  have quite strong attractions, which can consequently lead to  $\Delta K$  and  $\Sigma K$  quasi-bound states with binding energy of about 2 and 17 MeV, respectively. When the channel coupling of  $\Lambda K$  and  $\Sigma K$  is considered, the calculated phase shifts show a sharp resonance between the thresholds of  $\Lambda K$  and  $\Sigma K$  with spin-parity  $J^P = 1/2^-$  and width  $\Gamma \approx 5$  MeV. The strong attraction of  $\Sigma K$  and the sizeable off-diagonal matrix elements of  $\Lambda K$  and  $\Sigma K$  are responsible for the appearance of this resonance. Our further analysis reveal that the strong attractions of both  $\Delta K$  with  $I = 1$  and  $\Sigma K$  with  $I = 1/2$  dominantly come from the  $\sigma$  exchange and color-magnetic force of the one-gluon exchange (OGE), and the considerably large transition interaction from  $\Lambda K$  to  $\Sigma K$  are dominantly offered by the OGE. In other words, the OGE plays an important role in the  $\Delta K$ ,  $\Lambda K$ , and  $\Sigma K$  systems in the chiral SU(3) quark model study.

For low-energy hadron physics, it remains a controversial problem whether the gluon or the Goldstone boson is the proper effective degree of freedom besides the constituent quark. Glazman and Riska proposed that the Goldstone boson is the only other proper effective degree of freedom [4]. But Isgur gave a critique of the boson exchange model and insisted that the OGE governs the baryon structure [5]. Nonetheless, it is still a challenging problem in low-energy hadron physics whether OGE or vector-meson exchange is the right mechanism or both of them are important for describing the short-range quark-quark interaction.

In Refs. [6, 7], the chiral SU(3) quark model is extended to include the coupling between the quark and vector chiral fields. The OGE that dominantly governs the short-range quark-quark interaction in the original chiral SU(3) quark model is now nearly replaced by the

vector-meson exchange. By use of this model, we have obtained a satisfactory description of the  $NN$  and  $KN$  scattering phase shifts.

The purpose of this work is to perform a further dynamical study on the  $\Delta K$ ,  $\Lambda K$ , and  $\Sigma K$  systems in the extended chiral  $SU(3)$  quark model based on Refs. [2, 3]. Let's first briefly review the model (the detailed formula can be found in Ref. [7]). The total Hamiltonian of baryon-meson systems can be written as

$$H = \sum_{i=1}^5 T_i - T_G + \sum_{i < j=1}^4 V_{ij} + \sum_{i=1}^4 V_{i\bar{5}}, \quad (1)$$

where  $T_G$  is the kinetic energy operator for the c.m. motion, and  $V_{ij}$  and  $V_{i\bar{5}}$  represent the quark-quark and quark-antiquark interactions, respectively,

$$V_{ij} = V_{ij}^{OGE} + V_{ij}^{conf} + V_{ij}^{ch}, \quad (2)$$

where  $V_{ij}^{OGE}$  is the OGE interaction,  $V_{ij}^{conf}$  is the confinement potential, and  $V_{ij}^{ch}$  is the chiral fields induced effective quark-quark potential,

$$V_{ij}^{ch} = \sum_{a=0}^8 V_{\sigma_a}(\mathbf{r}_{ij}) + \sum_{a=0}^8 V_{\pi_a}(\mathbf{r}_{ij}) + \sum_{a=0}^8 V_{\rho_a}(\mathbf{r}_{ij}). \quad (3)$$

Here  $\sigma_0, \dots, \sigma_8$  are the scalar nonet fields,  $\pi_0, \dots, \pi_8$  are the pseudoscalar nonet fields, and  $\rho_0, \dots, \rho_8$  are the vector nonet fields. The expressions of all the interactions can be found in the literature [1, 2, 3, 7].

$V_{i\bar{5}}$  in Eq. (1) includes two parts: direct interaction and annihilation parts:

$$V_{i\bar{5}} = V_{i\bar{5}}^{dir} + V_{i\bar{5}}^{ann}, \quad (4)$$

with

$$V_{i\bar{5}}^{dir} = V_{i\bar{5}}^{conf} + V_{i\bar{5}}^{OGE} + V_{i\bar{5}}^{ch}, \quad (5)$$

and

$$V_{i\bar{5}}^{ch} = \sum_j (-1)^{G_j} V_{i\bar{5}}^{ch,j}. \quad (6)$$

Here  $(-1)^{G_j}$  represents the G parity of the  $j$ th meson. The  $q\bar{q}$  annihilation interactions,  $V_{i\bar{5}}^{ann}$ , are not included in this work because they are assumed not to contribute significantly to a molecular state or a scattering process, which is the subject of our present study.

All the model parameters are fixed before the calculation by some special constraints, such as the mass splits between  $N$ ,  $\Delta$  and  $\Lambda$ ,  $\Sigma$ , the stability conditions of  $N$ ,  $\Lambda$  and  $\Xi$ , and the masses of  $N$ ,  $\Sigma$ , and  $\overline{\Xi} + \overline{\Omega}$ . (For details See Refs. [1, 2, 3, 7].) Their values are listed in Table I, where the first set is for the original chiral SU(3) quark model, the second and third sets are for the extended chiral SU(3) quark model by taking  $f_{chv}/g_{chv}$  as 0 and 2/3, respectively. Here  $g_{chv}$  and  $f_{chv}$  are the coupling constants for vector coupling and tensor coupling of the vector meson fields, respectively.  $g_u$  and  $g_s$  are the OGE coupling constants and  $a^c$  represents the strength of the confinement potential. All these three sets of parameters can give a satisfactory description of the masses of the baryon ground states, the binding energy of the deuteron, and the  $NN$  scattering phase shifts.

From Table I one can see that for both set II and set III,  $g_u^2$  and  $g_s^2$  are much smaller than the values of set I. This means that in the extended chiral SU(3) quark model, the coupling constants of OGE are greatly reduced when the coupling of quarks and vector-meson field is considered. Thus the OGE that plays an important role of the quark-quark short-range interaction in the original chiral SU(3) quark model is now nearly replaced by the vector-meson exchange. In other words, the mechanisms of the quark-quark short-range interactions in these two models are quite different.

With all parameters determined in the extended chiral SU(3) quark model, the  $\Delta K$ ,  $\Lambda K$ , and  $\Sigma K$  states can be dynamically studied in the framework of the RGM, a well-established method for studying the interaction between two composite particles.

The  $\Delta K$  state has already been studied in Ref. [8], where the authors claimed that they find an attractive interaction in the  $\Delta K$  channel with  $L = 0$  and  $I = 1$ . This state has also been investigated in Ref. [9] based on the  $\chi$ -BS(3) approach. In Ref. [2], we study the  $\Delta K$  state in the chiral SU(3) quark model and find that the interaction of  $\Delta K$  with isospin  $I = 1$  is attractive. Such an attraction can make for a  $\Delta K$  quasi-bound state with about 2 MeV binding energy. Our further analysis shows that the attraction dominantly comes from the  $\sigma$  exchange and the color-magnetic force of OGE.

In this work, we further dynamically study the  $\Delta K$  state in the extended chiral SU(3) quark model, where the vector-meson exchanges play an important role in the short-range interaction. Figure 1 shows the diagonal matrix elements of the Hamiltonian in the generator coordinate method (GCM) [10] calculation, which can describe the interaction between two clusters  $\Delta$  and  $K$  qualitatively. In Fig. 1,  $s$  denotes the generator coordinate and  $V_{\Delta-K}$  is

TABLE I: Model parameters. The meson masses and the cutoff masses:  $m_{\sigma'} = 980$  MeV,  $m_{\kappa} = 980$  MeV,  $m_{\epsilon} = 980$  MeV,  $m_{\pi} = 138$  MeV,  $m_K = 495$  MeV,  $m_{\eta} = 549$  MeV,  $m_{\eta'} = 957$  MeV,  $m_{\rho} = 770$  MeV,  $m_{K^*} = 892$  MeV,  $m_{\omega} = 782$  MeV,  $m_{\phi} = 1020$  MeV, and  $\Lambda = 1100$  MeV.

	$\chi$ -SU(3) QM	Ex. $\chi$ -SU(3) QM	
	I	II	III
		$f_{chv} = 0$	$f_{chv} = 2/3g_{chv}$
$b_u$ (fm)	0.5	0.45	0.45
$m_u$ (MeV)	313	313	313
$m_s$ (MeV)	470	470	470
$g_u^2$	0.781	0.067	0.143
$g_s^2$	0.865	0.212	0.264
$g_{ch}$	2.621	2.621	2.621
$g_{chv}$		2.351	1.973
$m_{\sigma}$ (MeV)	595	535	547
$a_{uu}^c$ (MeV/fm <sup>2</sup> )	46.6	44.5	39.1
$a_{us}^c$ (MeV/fm <sup>2</sup> )	58.7	79.6	69.2
$a_{ss}^c$ (MeV/fm <sup>2</sup> )	99.2	163.7	142.5
$a_{uu}^{c0}$ (MeV)	-42.4	-72.3	-62.9
$a_{us}^{c0}$ (MeV)	-36.2	-87.6	-74.6
$a_{ss}^{c0}$ (MeV)	-33.8	-108.0	-91.0

the effective potential between the two clusters. From Fig. 1, one sees that the  $\Delta K$  state with isospin  $I = 1$  has an attractive interaction. Such an attraction can consequently make for a  $\Delta K$  bound state, and the binding energy is tabulated in Table II. As can be seen in Fig. 1, the  $\Delta K$  interaction for the isospin  $I = 1$  channel is more attractive in the extended chiral SU(3) quark model than that in the original chiral SU(3) quark model, and thus the cases II and III give much bigger binding energy than that of case I. In the original chiral SU(3) quark model, the  $\Delta K$  attraction comes from the  $\sigma$  exchange and the color-magnetic force of OGE. In the extended chiral SU(3) quark model, the OGE is nearly replaced by the

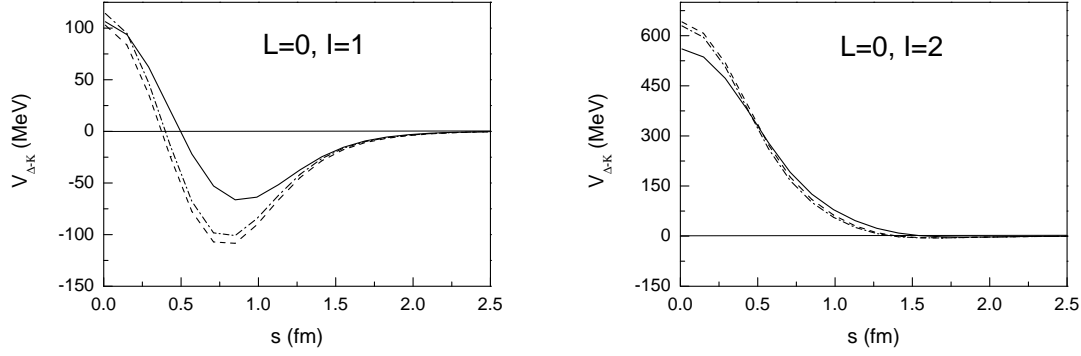


FIG. 1: The GCM matrix elements of the Hamiltonian. The solid curves represent the results obtained in the chiral SU(3) quark model. The dashed and dash-dotted curves show the results from the extended chiral SU(3) quark model by taking  $f_{chv}/g_{chv}$  as 0 and 2/3, respectively.

vector-meson exchanges and the attraction dominantly comes from the  $\sigma$  and  $\rho$  exchanges.

TABLE II: Binding energy of  $\Delta K$ .

Model	$B_{\Delta K}$ (MeV)	Attraction
I	3	OGE+ $\sigma$
II	20	$\sigma + \rho$
III	15	$\sigma + \rho$

Since the kaon meson is spin zero, the tensor force that plays an important role in reproducing the binding energy of the deuteron [6] now nearly vanishes in the  $\Delta K$  system. To examine whether  $(\Delta K)_{LSJ=0\frac{3}{2}\frac{3}{2}}$  is a possible resonance or bound state, the channel coupling between  $(\Delta K)_{LSJ=0\frac{3}{2}\frac{3}{2}}$  and  $(NK^*)_{LSJ=0\frac{3}{2}\frac{3}{2}}$  will be considered in future work.

The highlight that attracts our attention to the study of the  $\Lambda K$  system is the nucleon resonance  $S_{11}(1535)$ , of which the traditional picture is that of an excited three quark state, with one of the three quarks orbiting in an  $l = 1$  state around the other two [4, 11]. In contrast from the description in the constituent quark model (CQM), on the hadron level the  $S_{11}(1535)$  is argued to be a quasibound  $\Lambda K$ - $\Sigma K$  state [12, 13]. Nevertheless, in Ref. [14], the authors conclude that the  $S_{11}(1535)$  is not only generated by coupling to higher baryon-meson channels but appears to require a genuine three-quark component. So up to

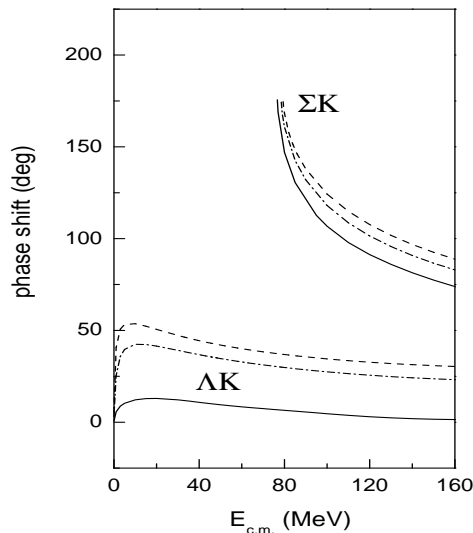


FIG. 2: The  $S$ -wave  $\Lambda K$  and  $\Sigma K$  phase shifts in the one-channel calculation. The notation is the same as in Fig. 1.

now the physical nature of the  $S_{11}(1535)$  — whether it is an excited three quark state or a quasi-bound baryon-meson  $S$ -wave resonance or a mixing of these two possibilities — is still a stimulating problem. A dynamical study on a quark level of the  $\Lambda K$  and  $\Sigma K$  interactions will undoubtedly make for a better understanding of the  $S_{11}(1535)$  and  $S_{11}(1650)$ .

TABLE III: Binding energy of  $\Sigma K$ .

Model	$B_{\Sigma K}$ (MeV)	Attraction
I	18	OGE+ $\sigma$
II	44	$\sigma + \rho + \phi$
III	33	$\sigma + \rho + \phi$

In Ref. [3], we study the  $\Lambda K$  and  $\Sigma K$  states in the chiral SU(3) quark model and find a strong attraction between  $\Sigma$  and  $K$ , which consequently results in a  $\Sigma K$  quasi-bound state with about 17 MeV binding energy. When the channel coupling of  $\Lambda K$  and  $\Sigma K$  is considered, a sharp resonance appears with spin-parity  $J^P = 1/2^-$ . Further analysis shows that the OGE plays an important role in the  $\Lambda K$  and  $\Sigma K$  systems.

In this work, we further study the  $\Lambda K$  and  $\Sigma K$  systems in the extended chiral SU(3) quark model where the OGE is nearly reduced. Figure 2 shows the  $\Lambda K$  and  $\Sigma K$  scattering

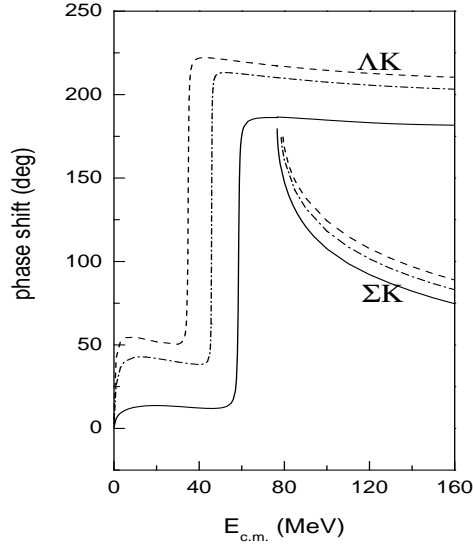


FIG. 3: The  $S$ -wave  $\Lambda K$  and  $\Sigma K$  phase shifts in the coupled-channel calculation. The notation is the same as in Fig. 1.

phase shifts in the one-channel calculation. The phase shifts denote that the  $\Sigma K$  state has a strong attractive interaction, which is consistent with the chiral Lagrangian calculation on the hadron level [12]. Such an attraction can result in a  $\Sigma K$  bound state, and the binding energy is tabulated in Table III. Similar to the  $\Delta K$  system, the interaction of  $\Sigma K$  is more attractive in the extended chiral SU(3) quark model than that in the original chiral SU(3) quark model, and thus model II and model III give much bigger binding energy than model I. In the original chiral SU(3) quark model, the  $\Sigma K$  attractive interaction comes from the  $\sigma$  exchange and the color-magnetic force of OGE. In the extended chiral SU(3) quark model, the OGE is nearly replaced by the vector-meson exchanges and the attraction dominantly comes from the  $\sigma$ ,  $\rho$ , and  $\phi$  exchanges.

TABLE IV: Mass and width of the  $\Lambda K$ - $\Sigma K$  resonance.

Model	Mass (MeV)	$\Gamma$ (MeV)
I	1670	$\approx 5$
II	1646	$\approx 4$
III	1655	$\approx 4$

We also consider the channel coupling of  $\Lambda K$  and  $\Sigma K$ , the phase shifts of which are

shown in Fig. 3. One sees that there is a sharp resonance between the thresholds of  $\Lambda K$  and  $\Sigma K$ . The narrow gap of the  $\Lambda K$  and  $\Sigma K$  thresholds, the strong attraction between  $\Sigma$  and  $K$ , and the sizeable off-diagonal matrix elements between  $\Lambda K$  and  $\Sigma K$  are responsible for the appearance of this resonance. The spin-parity of this resonance is  $J^P = 1/2^-$ , and its mass and width are tabulated in Table IV. The results from the extended chiral SU(3) quark model are quite similar to those from the original chiral SU(3) quark model, because  $\rho$  and  $\phi$  exchanges make contributions similar to OGE in this case. From the mass point of view and considering that the branching ratio of  $S_{11}(1650)$  to  $\Lambda K$  is  $3 - 11\%$  (With a partial width of about  $4.5 - 16.5$  MeV), the resonance we obtained seems to be an  $S_{11}(1650)$ , although the calculated width is a little bit small. To draw a final conclusion regarding what the resonance we obtained is and its exact theoretical mass and width, the effects of the  $s$ -channel  $q\bar{q}$  annihilation interaction as well as the coupling to the  $N\pi$ ,  $N\eta$ ,  $N\pi\pi$ , and even to the genuine  $3q$  component will be considered in future work.

In summary, we dynamically study the  $\Delta K$ ,  $\Lambda K$ , and  $\Sigma K$  states in the extended chiral SU(3) quark model, where the coupling between the quark and vector chiral fields are considered and thus the OGE is nearly reduced. Although the mechanisms of the quark-quark short-range interactions are quite different in the original chiral SU(3) quark model and the extended chiral SU(3) quark model, the theoretical results from these two models are very similar in these cases. They both show that the interactions of  $\Delta K$  with isospin  $I = 1$  and  $\Sigma K$  with isospin  $I = 1/2$  are attractive, which can consequently lead to  $\Delta K$  and  $\Sigma K$  quasibound states. When the channel coupling of  $\Lambda K$  and  $\Sigma K$  is considered, our calculated phase shifts show a sharp resonance between the thresholds of these two channels with spin-parity  $J^P = 1/2^-$ . Its exact theoretical mass and width await future work where more channel couplings and the decay properties will be studied.

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